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NOTE ON SHOCK REFLECTION COEFFICIENT

by

Nathan Gerber

August 1966

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BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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Exterior Ballistics Laboratory

RDT&E Project No. 1P222901A201

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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NGerber/sjw
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NOTE ON SHOCK REFLECTION COEFFICIENT

ABSTRACT

The shock reflection coefficient, R_w, is defined here as the ratio of pressure drop across a reflected disturbance from a shock wave to that across the incident disturbance wave (which is generated downstream of the shock). The treatment of shock reflection coefficients in the work of previous investigators is reviewed briefly. A derivation is given of a general expression for R_w in terms of shock curvature for nonequilibrium axisymmetric flow. Sample computations show the effects of nonequilibrium flow and three-dimensionality on the behavior of R_w. It is shown that R_w can become infinite in these situations, although for two-dimensional ideal gas flow its magnitude is always finite and usually very small.

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LIST OF SYMBOLS

f Nonequilibrium production term, Eq. (4)

K_w Shock wave curvature

M Local Mach number

n Arc length on curve normal to streamlines

p Pressure

q Flow speed

R_w Shock reflection coefficient

s Arc length on streamline

T Temperature

x Coordinate in direction of free stream

y Coordinate normal to free stream

β Shock wave angle

γ Ratio of specific heats

ϵ = 0 for two-dimensional flow; = 1 for axisymmetric flow.

η Arc length on left running Mach line

θ Direction of velocity vector

λ $= \beta - \theta$

μ Mach angle, = arc sin $(1/M)$

ξ Arc length on right running Mach line

ρ Density

Subscripts:

∞ Free stream value

w Value at shock wave

INTRODUCTION

The shock reflection coefficient, designated here by R_w, is a quantity of interest in the study of interaction of shock and Mach waves, particularly in connection with application of the shock-expansion method to computing supersonic flows. Essentially it provides a measure of the strength of a reflected disturbance from a shock wave relative to the strength of the incident disturbance wave (which is generated downstream of the shock).

Lighthill[1], Chu[2], and Chernyi[3*] have obtained the reflection coefficient for two-dimensional ideal gas flow. (See page 180 of Reference 3 for a brief discussion of errors in References 1 and 2.) Graphs of computations found in References 3 and 4 show that the magnitude of R_w is small except near the region of shock detachment, and that $|R_w|<1$ for finite Mach number. Note, however, that $R_w = -1$ for a straight shock and finite flow deflection, according to the definition in Equation (1). Eggers and Syvertson[5] consider a closely related quantity called "disturbance strength ratio." Their calculation for a case of air in vibrational equilibrium ($M_\infty = 10$, $T_\infty = 278^\circ$ K) indicates that caloric imperfections in the gas still leave the disturbance strength ratio small compared to unity if γ does not decrease appreciably below 1.3.

We now wish to investigate nonequilibrium effects on the shock reflection coefficient; here the shock curvature appears explicitly. In addition, we wish to examine the effect of three-dimensionality, after extending the definition of R_w to axisymmetric flow.

EQUATIONS

Refer to Figure 1; the shock reflection coefficient is defined as follows (as on page 232 of Reference 6):

$$R_w \equiv \lim_{\Delta\xi,\Delta\eta\to 0} \frac{p_C - p_B}{p_B - p_A} = \lim_{\Delta\xi,\Delta\eta\to 0} \frac{(dp/d\eta)_B\,\Delta\eta}{(dp/d\xi)_A\,\Delta\xi} \qquad (1)$$

* *Superscript numbers denote references which may be found on page 15.*

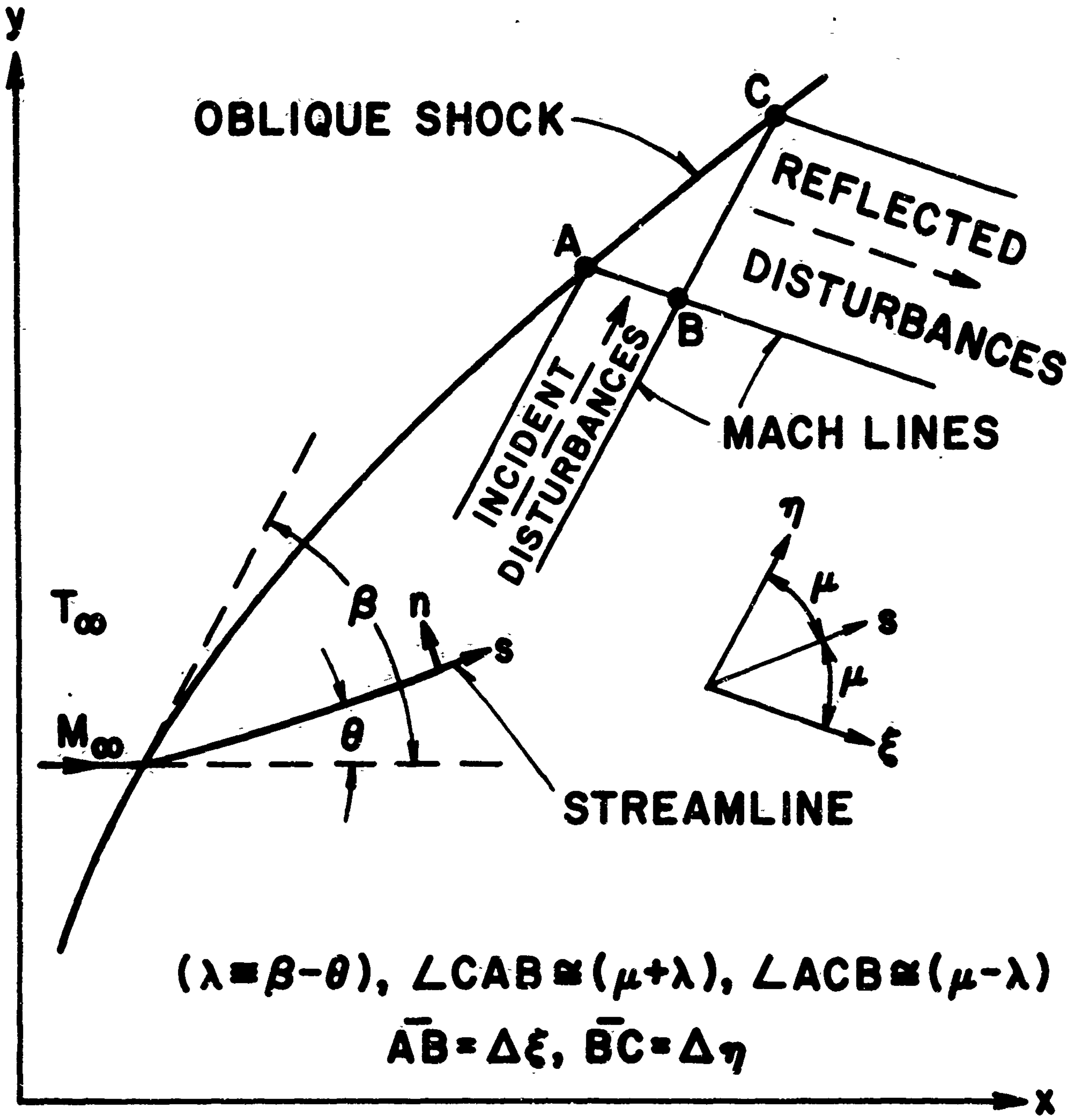

FIGURE 1. CROSS-SECTIONAL VIEW OF FLOW FIELD

Here the positive ξ and η directions make angles of $\mu(< 90^\circ)$ with the positive streamline direction. Applying the law of sines to Δ ABC, we get

$$R_w = \frac{(dp/d\eta)}{(dp/d\xi)} \frac{\sin(\mu+\lambda)}{\sin(\mu-\lambda)} \tag{2}$$

By means of the directional derivative relations

$$dp/d\eta = (dp/ds) \cos \mu + (dp/dn) \sin \mu$$

$$dp/d\xi = (dp/ds) \cos \mu - (dp/dn) \sin \mu$$

and the following relation valid at the shock wave:

$$(d/dn)_w = (\csc \lambda)_w \left[K_w \, d/d\beta - (\cos \lambda) \, d/ds\right]_w$$

the shock reflection coefficient is expressed as

$$R_w = - \frac{\sin(\mu+\lambda)}{\sin(\mu-\lambda)} \frac{\left[(\sin \mu)\frac{dp}{d\beta} K_w - \frac{dp}{ds} \sin(\mu-\lambda)\right]}{\left[(\sin \mu)\frac{dp}{d\beta} K_w - \frac{dp}{ds} \sin(\mu+\lambda)\right]} \tag{3}$$

and dp/ds is given by *

$$D \frac{dp}{ds} = (K_w \csc \lambda) \left(\frac{dp}{d\beta} \cot \lambda + \rho q^2 \frac{d\theta}{d\beta} \right) + \frac{\varepsilon}{y} \rho q^2 \sin \theta + \rho q^2 f \tag{4}$$

where $D = 1 - M^2 + \cot^2\lambda$, and f is the nonequilibrium "production term" ($f = 0$ for frozen or equilibrium flow); $\epsilon = 0$ or 1 for two-dimensional or axisymmetric flow, respectively.

CALCULATIONS

We consider dissociating air, using the model presented in Reference 7, as an example of nonequilibrium effects on R_w; for this flow the air is in vibrational equilibrium at the shock. A typical set of results is shown in Figure 2 for a wide range of shock curvature. It is seen that for each value of K_w there is a critical deflection angle producing infinite R_w; this results from the vanishing of $dp/d\xi$ in Equation (2).

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* *This formula is found in section 5 of Reference 7, where a typographical error occurs; the multiplication sign should be replaced by a plus sign.*

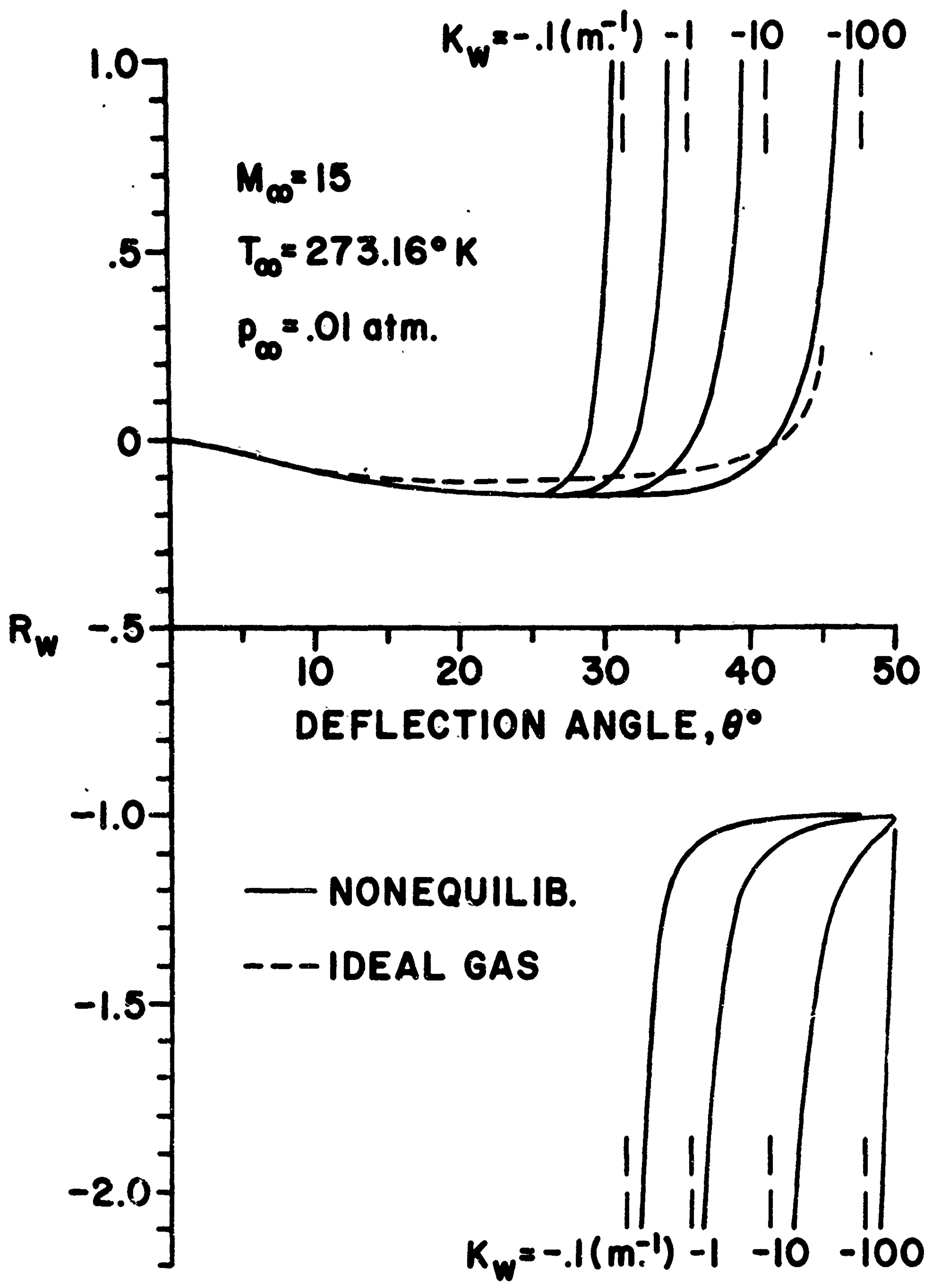

FIGURE 2. SHOCK REFLECTION COEFFICIENT FOR NONEQUILIBRIUM DISSOCIATING AIR, TWO DIMENSIONAL FLOW

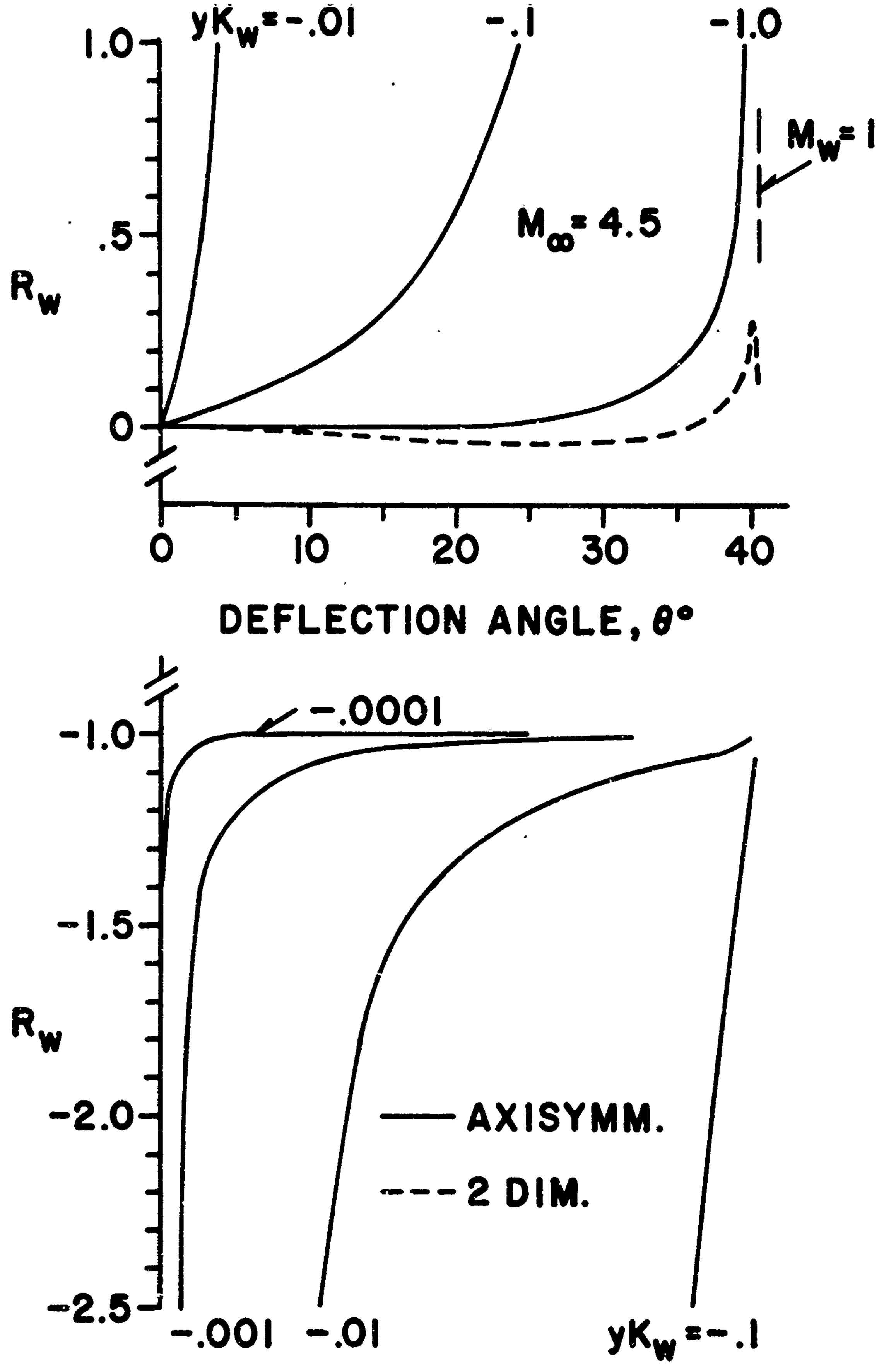

FIGURE 3. SHOCK REFLECTION COEFFICIENT FOR AXISYMMETRIC FLOW, IDEAL GAS

For deflections smaller than this critical value the behavior of R_w is similar to that for ideal gas flow except for the sharp rise at the asymptote; beyond the asymptote R_w approaches - 1 at the detachment angle.

When ϵ is taken equal to 1 in Equation (4), the definition of shock reflection coefficient [(Equation (1)] can be applied to axisymmetric flow. Figure 3 shows some typical results in ideal gas for several values of the parameter yK_w. (Generally for flows over smooth ogival bodies of revolution, $-0.1 < yK_w \leq 0$.) The behavior of R_w here differs considerably from that for two-dimensional flow. There is a critical deflection angle for each yK_w and a limiting R_w value of - 1 as detachment angle is approached. Formally, $yK_w = -\infty$ corresponds to two-dimensional flow, and the figure shows the approach of the curves with decreasing yK_w to the two-dimensional one.

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REFERENCES

1. Lighthill, M. J. The Flow Behind a Stationary Shock. Phil. Mag., 40, 214-220 (1949).

2. Chu, B. T. On Weak Interaction of Strong Shock and Mach Waves Generated Downstream of the Shock. J. Aero. Sci., 19, 433-446 (1952).

3. Chernyi, G. G. Introduction to Hypersonic Flow. Academic Press, New York, (1961) (Translated and edited by R. F. Probstein)

4. Waldman, G. D. and Probstein, R. F. An Analytic Extension of the Shock-Expansion Method. J. Aerospace Sci., 28, 119-132 (1961).

5. Eggers, A. J., Jr. and Syvertson, C. A. Inviscid Flow about Airfoils at High Supersonic Speeds. NACA TN 2646 (1952). Superseded by NACA TR 1123 (1953).

6. Eggers, A. J., Jr.; Savin, R. C. and Syvertson, C. A. The Generalized Shock-Expansion Method and Its Application to Bodies Traveling at High Supersonic Air Speeds. J. Aero. Sci., 22, 231-238 (1955).

7. Spurk, J. H.; Gerber, N. and Sedney, R. Characteristic Calculation of Flowfields with Chemical Reactions. AIAA Jour., 4, 30-37 (1966).

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The shock reflection coefficient, R_{AW}, is defined here as the ratio of pressure drop across a reflected disturbance from a shock wave to that across the incident disturbance wave (which is generated downstream of the shock). The treatment of shock reflection coefficients in the work of previous investigators is reviewed briefly. A derivation is given of a general expression for R_{AW} in terms of shock curvature for nonequilibrium axisymmetric flow. Sample computations show the effects of nonequilibrium flow and three-dimensionality on the behavior of R_{AW}. It is shown that R_{AW} can become infinite in these situations, although for two-dimensional ideal gas flow its magnitude is always finite and usually very small.

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